

Family Name: \_\_\_\_\_ Given Name: \_\_\_\_\_ I.D.# \_\_\_\_\_

## MAT3320 Assignment 2

**Total: 10 marks. Due date: Tuesday, June 6, on or before 4:00pm.**

In MATH Department (585 King Edward), there is a Drop-Box. You need to put your assignment into the box **on or before 4:00pm** on the due date. Late assignments will not be accepted.

1. (7 marks=1+1+2+3) Consider the following equation:

$$2xy'' + y' + 3y = 0.$$

- (a) Show that  $x_0 = 0$  is a regular singular point.
- (b) Write down the indicial equation and solve it to determine  $r_1$  and  $r_2$ ,  $r_1 \geq r_2$ .
- (c) Let  $y = \sum_{n=0}^{\infty} c_n(r)x^{n+r}$ . Determine the recursive relation for  $c_n(r)$ , i.e., relation between  $c_{n+1}(r)$  and  $c_n(r)$ .
- (d) Take  $c_0(r) = 1$ . Find two linearly independent solutions  $y_1$  and  $y_2$  which are valid for  $x > 0$  near  $x_0 = 0$ .

2. (1 mark) Consider the differential equation

$$4x^2y'' + 4xy' + (3x - 36)y = 0, \quad x > 0. \tag{1}$$

By letting  $z = \sqrt{x}$ , the equation can be changed to the following Bessel's equation

$$z^2y_z'' + zy_z' + (3z^2 - 36)y = 0, \quad z > 0.$$

Find the general solution of the equation (1).

3. (2 marks) Consider the Sturm-Liouville problem

$$y'' + 4y' + \lambda y = 0, \quad 0 < x < \pi, \quad y'(0) = 0, y'(\pi) = 0.$$

Is  $\lambda = 8$  an eigenvalue? If yes, find the corresponding eigenfunction; if no, explain why.